ABSTRACT

Iterative Bayesian image reconstruction algorithms for emission computerized tomography have been derived and applied using three general classes of a priori source distribution constraints: 1) maximum entropy constraints on source elements; 2) source continuity and boundary information; 3) ideal and fuzzy pattern source distribution constraints. This work has recently been extended to include a combined Bernoulli process and entropy analysis on source element strength correlations and also to provide for more general a priori probability distributions of probable source element strengths in subsets of a source field. Improvement in image reconstruction over standard methods has been demonstrated by applying these Bayesian algorithms to computer simulation data and experimental nuclear isotope imaging data.

INTRODUCTION

Incorporating a priori source information in Bayesian formalism for image reconstruction has recently attracted more attention [1-3]. We have proposed a Bayesian image processing (BIP) iterative BIP algorithms using several different classes of a priori source distribution constraints [4-11]. This work has now been extended to include a statistical Bernoulli process of source elements with fixed total number of photon emissions and a maximum entropy analysis on individual source element probability distributions. It has also been extended to provide for more general a priori probability distributions of probable source element strengths in subsets. Such a priori source information is formulated in terms of probability density functions of source element strengths and spatial correlations. The corresponding iterative BIP algorithms are derived by maximizing the Bayesian functions via the expectation-maximization technique and assuming data obey Poisson statistics [12-14]. Improved results obtained by applying the algorithms to computer generated data and experimental phantom imaging data are presented.

A PRIORI SOURCE INFORMATION FUNCTIONS

As shown in reference [6], maximum entropy considerations result in a Poisson functional form of probability density function \( p_i(\phi_j) \) for each source element (voxel) strength \( \phi_j \), under the assumption of the existence of a defined mean \( \phi_j \), and a defined variation \( \sigma_j^2 \). Where \( \phi_j \) is random variable \( \phi_j \geq 0 \), i.e.,

\[
p_i(\phi_j) = e^{-\phi_j} (\phi_j^{\alpha_j} \sigma_j^{-\alpha_j} - 1)
\]  

(1)

In nuclear isotope imaging applications, function \( p_i(\phi_j) \) stands for the a priori probability distribution of the photon emission from voxel \( j \) per unit time at time \( t = 0 \).

If all voxel values \( \{\phi_j\} = \Phi \) are uncorrelated with each other, then the a priori probability density function of the source distribution \( \Phi \) is:

\[
P_j(\Phi) = \prod_{j=1}^{N} p_j(\phi_j) = \prod_{j=1}^{N} e^{-\phi_j} (\phi_j^{\alpha_j} \sigma_j^{-\alpha_j} - 1)
\]  

(2)

where \( j \) is the number of voxels.

If the total number \( N \) of emitted photons is approximately fixed (corresponding hypothetically to a given patient dose and an extended count time), all voxel values are no longer strictly uncorrelated with each other. The distribution of the \( N \) photons over the \( J \) voxels can then be assumed to obey a Bernoulli process [15], whence:

\[
P_j(\Phi) = \frac{N!}{\prod_{j=1}^{J} \phi_j!} \prod_{j=1}^{J} \left( \frac{\phi_j}{h_j} \right)^{h_j} \prod_{j=1}^{J} \left( 1 - \frac{\phi_j}{h_j} \right)^{h_j - \phi_j}
\]  

(3)

where each voxel value has the Poissonian functional probability distribution (1) and the sum of \( \{\phi_j\} \) is proportional to the total number \( N \), \( h_j = \sum \phi_j \) is a normalization constant.

Note that although the two source probability functions \( P_j(\Phi) \) and \( P_j(\Phi) \) reflect two different physical modeling processes, both of them assume that each voxel value \( \phi_j \) has a defined mean \( \phi_j \) and a defined variation without knowing their values.

In some cases, a voxel value in an a priori sub-set of source field can be more restrictively characterized by a given probable mean value and variation. It will then have a set of discrete probabilities of falling into each of the discrete gray levels reflecting the intensity resolution of the imaging system. Such a priori probability information of source distributions around a set of probable source strengths can be expressed by the probability density functions (4), (5) and (7) below.

As outlined in reference [7], if the strengths of source elements can be divided into \( k + 1 \) distinct gray levels of \( \{\Delta_i\} \), \( i = 0, 1, \ldots, k - 1 \), and each voxel value \( \phi_j \) has different probabilities of falling into each of the gray levels, then

\[
p_j(\phi_j) = \sum_{i=1}^{k} C_i e^{-\phi_j \Theta} (\phi_j)^{\alpha_i} \sigma_j^{-\alpha_i}
\]  

(4)

where the last term can have a Poisson rather than a Gaussian functional form to exclude negative source strengths for voxels associated with the lowest gray level \( \phi_j \) and the \( C_i \) and \( \Theta \) are the normalization constants reflecting the anticipated average number of voxels falling within corresponding gray levels \( \{\Delta_i\} \) and \( \phi_j \).

If there is no spatial correlation information available, the probability density function of the source distribution is:

\[
P_j(\Phi) = \prod_{j=1}^{J} p_j(\phi_j)
\]  

(5)

\[
P_j(\Phi) = \prod_{j=1}^{J} \sum_{i=1}^{k} C_i e^{-\phi_j \Theta} (\phi_j)^{\alpha_i} \sigma_j^{-\alpha_i}
\]  

(6)

With spatial correlations of source element strengths available, the source distribution can be viewed as a patterned source distribution. Assume as one possibility that the spatial correlation function \( w(i) \) has the Gaussian functional form:

\[
w(i) = a e^{-b \delta^2}
\]  

then [8]

\[
P_j(\Phi) = \prod_{j=1}^{J} \left[ \sum_{i=1}^{k} C_i e^{-\phi_j \Theta} (\phi_j)^{\alpha_i} \sigma_j^{-\alpha_i} \right]
\]  

(7)

where \( a, b \) and \( \delta \) are constants, index \( i \) sums over different probable pattern configurations.

The four a priori source probability information functions \( P(\Phi), P_j(\Phi), P_j(\Phi) \), and \( P_j(\Phi) \) are presented above in the order of \( j \) from least to most restrictive. They can be applied to
correspondingly suitable situations. Each of these functions will be used to derive the iterative BIP algorithms respectively assuming Poisson statistics in the following section.

**Iterative BIP Algorithms**

Bayesian image processing offers the potential to yield a maximum likelihood source distribution \( \Phi \) incorporating any a priori source information which may be available. Mathematically, this is expressed by the Bayes' Law:

\[
P(\Phi | Y) = \frac{P(Y | \Phi) P(\Phi)}{P(Y)}
\]

and \( \Phi \) maximizes the In Bayesian function \( \ln P(\Phi | Y) \)

\[
\frac{\partial}{\partial \Phi} \ln P(\Phi | Y) |_{\Phi = \Phi^*} = 0
\]

where the likelihood probability density function \( P(Y | \Phi) \) reflects the statistical nature of measured data \( Y \) from source distribution \( \Phi \), the a priori source probability density function \( P(\Phi) \) was discussed in the previous section and the data probability density function \( P(Y) \) is not related to the maximal value of \( P(\Phi | Y) \) and so will be omitted.

If there is no a priori source information available, \( P(\Phi) \) can be assumed as constant, implying that all possible source distributions are equally likely; maximizing the Bayesian function results in the same solution as the expectation-maximization technique [12-14], among many other iterative schemes [16-17], respectively will be applied to computer generated data and experimental phantom imaging data for image reconstruction in the next section.

**Simulation and Experimental Results**

Computer simulation is carried out in three steps. First a two dimensional source distribution [15] is specified, as in Fig.1, with two point sources of strength 105, separated by 8 voxel units, superimposed upon a background of strength 1 source unit. The point spread function PSF is calculated from an assumed functional form:

\[
R_j \approx e^{-b |D|/2 - j^2}
\]

where \( T = \text{FWHM}/2 = 4 \) voxel units, a rather poor resolution imaging system.

Second, the data distribution \( Y_j \) is calculated using the relation

\[
Y_j = \sum R_j S_j
\]

as shown in Fig.2. Note that here \( Y_j \) are ideal data. The effects of Poisson randomization are studied using experimental phantom imaging data.
The last step is to apply the four algorithms to the simulation data \( \{Y, l\} \), where the initial estimate can be assumed, for simplicity, as:

\[
\phi^{(0)} = Y / \sum R \delta . 
\]  

(22)

Since algorithms (16) and (17) are derived on the assumption of each voxel value \( \phi \), having a defined mean \( \phi \), an optimal choice of \( \phi \) is to let \( \{\phi_j, S_j\} \). Under this condition, algorithms (16) and (17) generate almost identical results to that of Fig.1 in less than 5 iterations.

Since \( \{S_j\} \) is of course generally unknown, the choice of \( \{\phi_j = S_j\} \) is unavailable in most situations. A more realistic and weakly restrictive choice was therefore usually employed:

\[
\phi_j = \phi_j^{(n)} + \tau d_j^{(n)} = \phi_j^{(n)} + (\lambda + \tau) d_j^{(n)} . 
\]  

(23)

If the step factor \( \tau = 0 \), the mean of voxel \( k \) is the update iterated voxel value. In this case, algorithm (16) approaches the ML result [13-14, 46]. With \( \tau > 0 \), the mean value advances in the direction \( d_j^{(n)} \). As usual, \( \tau = 1 \). Figs. 3 and 4 show the results of algorithms (16) and (17) respectively after 100 iterations for the simulation data (21), where the constants \( A = 1.0, B = L^2, \kappa = 2 \), and \( L \) is the number of iterations.

Experimental phantom imaging data were obtained by using a Picker Dyna Gamma Camera without a collimator and imaging two parallel catheters containing a solution Co-57 in a plane parallel to the camera face. The output from the camera was arranged in a 32x32 matrix as shown in Fig.6 with the two lines of tubing oriented in the \( y \)-direction.
The PSF is the camera output imaging a point source of Co$^{60}$ at the same depth as the two lines of tubing and is arranged in a 32x32 matrix as shown in Fig.7. The FWHM = 9 pixel units is very close to the separation of the two lines of tubing. Since the camera system can be assumed to be approximately spatially invariant, $R_j$ is then formed from PSF [18].

The initial estimate $\Phi_0$ is calculated using Eq.(22) with the measured $\{Y, l\}$ and $\{R_j\}$. Figs 8 and 9 are the results of algorithms (16) and (17) respectively after 25 iterations for the phantom imaging data. The ML algorithm generates a result, shown in Fig.10, after 50 iterations.

The estimate values $\phi_0 = 8, \phi_1 = 105, \phi_2 = 115$ and $l = 7$ were used in carrying out the results of algorithms (18) and (19) for the phantom imaging data. Figs.11 and 12 show the results of algorithms (18) and (19) respectively after 25 iterations.

The comparisons of the results of using BIP algorithms and the ML algorithm for the computer simulation and experimental phantom imaging data demonstrate the improvement in image processing if valid a priori source information of (5) and (7) is available or the general source distribution modeling functions (2) and (3) are used.

**DISCUSSION**

The BIP formalism has the advantage of considering both the statistical behavior of the measured data and a priori source distribution information. In this paper and previous work [4-11], we focused on the a priori source element strength and spatial correlation information. In emission computerized tomography, the tissue attenuation and scattering may be also an example of a priori source information. There are two ways to handle the attenuation and scattering correction, one is to incorporate the correction into the values $\{R_j\}$ and another is into the probability density function $P(\Phi)$. As indicated by function (11) and [6], $R_j$ include the effects of radioisotope decay, tissue attenuation and scattering. Specifically, the effect of radioisotope decay decreases the photon emission gradually as a factor $e^{-\alpha l}$ during the scanning period, where $\alpha$ is the decay constant and $l$ the time at which the data element $Y_j$ is being measured.

The attenuation effect influences PET and SPECT in different ways [16, 19-20].

1. for PET, the attenuation factor is $exp\{ - \int_0^l \mu(Y_i) \, dl \}$, where $Y_i$ is the path coordinate of projection ray $Y_i$, $l_i$ the path length of $Y_i$ intersecting the patient body, and $\mu$ the absorption coefficient. Since the factor is independent of the voxel index $j$, so the mean value of $Y_j$ becomes by analogy with prior work [19]:

$$\sum_j R_j \phi_j = \exp\{ - \int_0^l \mu(Y_i) \, dl_i - \alpha \} \sum_j R_j \phi_j$$

where $R_j \phi_j$ depends only upon the projection-voxel geometry and scattering. The exponential term of Eq.(25) can be calculated easily prior to image reconstruction for each projection ray $Y_i$ if the absorption coefficient $\mu$ and body contour are known. Note that the attenuation correction of the mean value of (25) is different from those of the projection value correction [19, 21].
for SPECT, the attenuated mean value of $Y$, is:

$$e^{-\mu L_i} \sum_j R_{ij} \phi_j \exp \left( - \int u(l_i) \,dl_i \right)$$

(26)

where $L_i$ is the path length of photons from voxel $j$ to the surface of the patient body along projection ray $Y$, and $l_i$ is the position of voxel $j$ on $Y$.

Somewhat more exact means of calculating scattering and attenuation corrections are currently being formulated using the Compton and Rayleigh scattering formulas [24-25]. The work is in progress on incorporating formulas (25), (26), and scattering effect and attenuation correction of $P(\Omega)$ into Bayesian formalism.

REFERENCES


